2023 年度 数理論理学 復習問題 (7)

問題 1 以下の命題論理式の証明図を示し，対応する擬似証明コードを書け.

(1) $\mathtt{P} \to (\mathtt{P} \to \mathtt{Q}) \to \mathtt{Q}$

(2) $(\mathtt{Q} \to \mathtt{R}) \to \mathtt{Q} \wedge \mathtt{P} \to \mathtt{R}$

(3) $(\mathtt{Q} \to \mathtt{Q} \to \bot) \to \neg\mathtt{Q}$

(4) $(\mathtt{Q} \to \mathtt{P}) \to (\mathtt{P} \vee \mathtt{Q}) \to \mathtt{P}$

(5) $(\mathtt{P} \to \mathtt{Q}) \to (\mathtt{P} \wedge \mathtt{Q} \leftrightarrow \mathtt{P})$

2023 年度 数理論理学 復習問題解答 (7)

問題 1

(1)

証明図:

$$\dfrac{\dfrac{\dfrac{[\mathtt{P} \to \mathtt{Q}]^1 \quad [\mathtt{P}]^2}{\mathtt{Q}} \to\mathrm{E}}{(\mathtt{P} \to \mathtt{Q}) \to \mathtt{Q}} \to\mathrm{I}^1}{\mathtt{P} \to (\mathtt{P} \to \mathtt{Q}) \to \mathtt{Q}} \to\mathrm{I}^2$$

擬似証明コード (解答例):

```
assume 1 : P → Q and 2 : P
hence Q by →E
hence P → (P → Q) → Q by →I[1, 2]
```

(2)

証明図:

$$\dfrac{\dfrac{\dfrac{[\mathtt{Q} \to \mathtt{R}]^2 \quad \dfrac{[\mathtt{Q} \land \mathtt{P}]^1}{\mathtt{Q}} \land\mathrm{E}}{\mathtt{R}} \to\mathrm{E}}{\mathtt{Q} \land \mathtt{P} \to \mathtt{R}} \to\mathrm{I}^1}{(\mathtt{Q} \to \mathtt{R}) \to \mathtt{Q} \land \mathtt{P} \to \mathtt{R}} \to\mathrm{I}^2$$

擬似証明コード (解答例):

```
assume 1 : Q → R and 2 : Q ∧ P
from 2 have Q by ∧E
hence R using 1 by →E
hence (Q → R) → Q ∧ P → R by →I[2, 1]
```

(3)

証明図:

$$\dfrac{\dfrac{\dfrac{[\mathtt{Q} \to \mathtt{Q} \to \bot]^2 \quad [\mathtt{Q}]^1}{\mathtt{Q} \to \bot} \to\mathrm{E} \quad [\mathtt{Q}]^1}{\dfrac{\bot}{\neg\mathtt{Q}} \neg\mathrm{I}^1} \to\mathrm{E}}{(\mathtt{Q} \to \mathtt{Q} \to \bot) \to \neg\mathtt{Q}} \to\mathrm{I}^2$$

擬似証明コード (解答例):

```
assume 1 : Q → Q → ⊥ and 2 : Q
hence ⊥ by →E
hence ¬Q by ¬I[2]
hence (Q → Q → ⊥) → ¬Q by →I[1]
```

(4)
証明図:

$$\dfrac{\dfrac{\dfrac{[\mathtt{P}\lor\mathtt{Q}]^2 \quad [\mathtt{P}]^1 \quad \dfrac{[\mathtt{Q}\to\mathtt{P}]^3 \quad [\mathtt{Q}]^1}{\mathtt{P}}\to\mathrm{E}}{\mathtt{P}}\lor\mathrm{E}^1}{(\mathtt{P}\lor\mathtt{Q})\to\mathtt{P}}\to\mathrm{I}^2}{(\mathtt{Q}\to\mathtt{P})\to(\mathtt{P}\lor\mathtt{Q})\to\mathtt{P}}\to\mathrm{I}^3$$

擬似証明コード (解答例):

```
assume 1 : P ∨ Q and 2 : Q → P
show P as follows:
  distinguish cases using 1
  case P
    hence P trivially
  case Q
    hence P using 2 by →E
  hence claim by ∨E
hence (Q → P) → (P ∨ Q) → P by →I[1, 2]
```

(5)
証明図:

$$\dfrac{\dfrac{\dfrac{[\mathtt{P}\land\mathtt{Q}]^1}{\mathtt{P}}\land\mathrm{E} \quad \dfrac{[\mathtt{P}]^1 \quad \dfrac{[\mathtt{P}\to\mathtt{Q}]^2 \quad [\mathtt{P}]^1}{\mathtt{Q}}\to\mathrm{E}}{\mathtt{P}\land\mathtt{Q}}\land\mathrm{I}}{\mathtt{P}\land\mathtt{Q}\leftrightarrow\mathtt{P}}\leftrightarrow\mathrm{I}^1}{(\mathtt{P}\to\mathtt{Q})\to(\mathtt{P}\land\mathtt{Q}\leftrightarrow\mathtt{P})}\to\mathrm{I}^2$$

擬似証明コード (解答例):

```
assume 1 : P → Q
show P ∧ Q ↔ P as follows:
  (=>)
    assume 2 : P ∧ Q
    hence P by ∧E
  (<=)
    assume 3 : P
    hence Q using 1 by →E
    hence P ∧ Q using 3 by ∧I
  hence claim by ↔I[2, 3]
hence (P → Q) → (P ∧ Q ↔ P) by →I[1]
```